Documentation and Validation of EveryCalc's Transmission Strength Tool

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Abstract

Making sure a gearbox is designed to handle the stresses that are inflicted upon it can be a tedious and repetative task, but is rather formulaic, making it an excellent candidate for an automated tool with a flexible frontend.

1 Strength of Gears

There are many failure points on a gear, but the most common and the only one that can be fairly analyzed without knowing the exact geometry (such as pocketing, shaft interface geometry) is that of the gear tooth. Engineer's Edge explains the common way of calculating tooth strength by considering the load as being fully transmitted by one tooth which is a beam in bending.

$$W_t = \frac{SwY(N,\alpha)}{D_p}, \tag{1}$$

where W_t is the maximum allowable tangential force on the gear tooth, S is the maximum allowable stress in the gear, w is the width of the tooth, Y is the *Lewis Factor*, and D_p is the diametral pitch (not the module, which is the reciprocal of the diametral pitch).

To determine the torque-carrying capacity of the gear, we substitute in an expression for torque T,

$$T = W_t\ r = W_t \frac{N}{2\ D_p} \tag{2}$$

$$\frac{2\ D_p}{N}\ T = \frac{S\ w\ Y(N,\alpha)}{D_p} \tag{3}$$

$$T_{max,gear} = \frac{S_{gear}\ w\ Y(N,\alpha)\ N}{2\ D_p^2} \tag{4}$$

The Lewis Factor Y is obtained by 1-D interpolation.

LEWIS FACTOR - Y

NO. OF TEETH	14 1/2° INVOLUTE	20° INVOLUTE
10	0.176	0.201
11	0.192	0.226
12	0.210	0.245
13	0.223	0.264
14	0.236	0.276
15	0.245	0.289
16	0.255	0.295
17	0.264	0.302
18	0.270	0.308
19	0.277	0.314
20	0.283	0.320
22	0.292	0.330
24	0.302	0.337
26	0.308	0.344
28	0.314	0.352
30	0.318	0.358
32	0.322	0.364
34	0.325	0.370
36	0.329	0.377
38	0.332	0.383
40	0.336	0.389
45	0.340	0.399
50	0.346	0.408
55	0.352	0.415
60	0.355	0.421
65	0.358	0.425
70	0.360	0.429
75	0.361	0.433
80	0.363	0.436
90	0.366	0.442
100	0.368	0.446
150	0.375	0.458
200	0.378	0.463
300	0.382	0.471
RACK	0.390	0.484

Figure 1: Lewis Factor values, tabulated

S_{gear} will be considered to be the tensile yield strength.

Observations: to make a gear stronger, increasing its width or base material strength will have a linear benefit. Increasing the number of teeth will have a hyperlinear benefit (as it influences the lewis factor). Using a lower pressure angle will help the lewis factor. Using a lower diametral pitch (a coarser gear) will also improve strength.

2 Strength of Shafts

Shafts are considered to be in pure torsion. This means that they experience stress that can be computed as

$$\sigma_{shear,outside} = \frac{Tr}{J} = \frac{Td}{2J} \tag{5}$$

Solving for the torque and substituting in maximum allowable shear stress S_{shaft} for σ yields

$$T_{max,shaft} = S_{shaft}\frac{J}{r} \tag{6}$$

S_{shaft} will be the maximum shear stress, or the tensile yield stress divided by two.

3 Strength of Timing Belt Runs

Belt strength is calculated from the tables in the Gates Light Power and Precision Manual.

predetermined number of grooves, pitch diameters and rpm's. These ratings must be multiplied by the appropriate width factor and applicable belt length factor to obtain the corrected torque rating. (See Step 4 of Drive Selection Procedure on Page 24.)

Belt Width (mm)	9	15	20	25
Width Multiplier	0.60	1.00	1.33	1.67

Rated Torque (lb-in) For Small Sprocket - 15mm Belt Width*

Number of Grooves	18	20	22	24	26	28	32	36	40	45	50	56	62	74	80
Pitch Diameter (mm)	28.65	31.83	35.01	38.20	41.38	44.56	50.93	57.30	63.66	71.62	79.58	89.13	98.68	117.77	127.32
Pitch Diameter (in)	**1.128**	**1.253**	**1.379**	**1.504**	**1.629**	**1.754**	**2.005**	**2.256**	**2.506**	**2.820**	**3.133**	**3.509**	**3.885**	**4.637**	**5.013**
10	78.24	93.61	109.00	124.20	139.30	154.3	184.3	214.10	243.6	280.3	316.6	359.9	403.0	488.3	530.6
20	72.38	87.11	101.80	116.40	130.90	145.2	173.9	202.40	230.6	265.6	300.4	341.7	382.8	464.2	504.6
40	66.53	80.60	94.69	108.60	122.40	136.1	163.5	190.70	217.6	251.0	284.1	323.5	362.7	440.2	478.5
60	63.11	76.80	90.51	104.00	117.50	130.8	157.5	183.90	209.9	242.4	274.6	312.9	350.9	426.1	463.3
100	58.80	72.01	85.23	98.27	111.20	124.1	149.8	175.20	200.4	231.7	262.6	299.5	336.0	408.3	444.1
200	52.94	65.51	78.08	90.46	102.80	115.0	139.4	163.50	187.4	217.0	246.3	281.2	315.8	384.2	418.1
300	49.52	61.70	73.89	85.90	97.83	109.7	133.3	156.70	179.7	208.4	236.8	270.6	304.0	370.1	402.8
400	47.09	59.00	70.92	82.65	94.31	105.9	129.0	151.80	174.3	202.3	230.0	263.0	295.6	360.0	391.8
500	45.20	56.91	68.61	80.14	91.59	103.0	125.6	148.00	170.1	197.6	224.7	257.0	289.0	352.1	383.3
600	43.66	55.19	66.73	78.08	89.36	100.6	122.9	144.90	166.7	193.7	220.4	252.2	283.6	345.6	376.2

Length Correction Factor			0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15	1.20	1.25	1.30
For Belt Length	From	Length (mm)	200	215	260	315	375	450	540	650	780	935	1130	1355	1625	1960
		# of teeth	40	43	52	63	75	90	108	130	156	187	226	271	325	392
	To	Length (mm)	210	255	310	370	445	535	645	775	930	1125	1350	1620	1955	2000
		# of teeth	42	51	62	74	89	107	129	155	186	225	270	324	391	400

Shaded area indicates drive conditions where reduced service life can be expected. Contact Gates Product Application Engineering for specific recommendations.

Figure 2: Exemplary data from the Gates manual.

These tables list allowable pulley torque $T(\omega, N)$ as a function of RPM ω and pulley teeth N. Note that 6 teeth should be in engagement. 2-D interpolation is used to determine values on the in-betweens. Tabulated values outside the bounds are extrapolated. Omitted values are presumed to be zero.

$$T_{base} = \text{interp2D}(\{N\}, \{\omega\}, [T], N_{sprocket}, \omega_{sprocket}) \tag{7}$$

$$K_{length} = \text{interp1D}(\{L\}, \{K_{lengths}\}, \{L_{belt}\}) \tag{8}$$

$$K_{width} = \text{lookup}(\{w\}, \{K_{widths}\}, \{w_{belt}\}) \tag{9}$$

$$T_{rated,\ sprocket} = K_{length} \times K_{width} \times T_{base} \tag{10}$$

It should also be noted that the number of teeth engaged with the pulley is recommended to be no less than 6.

4 Strength of COTS Planetaries

4.1 VexPro VersaPlanetary

VexPro's VersaPlanetaries come with a Load Rating Guide.

The key failure points identified are:

- 10:1, 9:1, and 7:1 stages have a torque capacity of **100 N-m**.
- Ratchet slices have a torque capacity of **160 N-m**.
- 1/2" hex output shafts fail at **157 N-m**.
- 1/2" round output shafts fail at **130 N-m**.
- 3/8" hex output shafts fail at **57 N-m**.
- CIM-style output shafts fail at **29 N-m**.

These ratings, as this calculator, do not take into consideration bending loads which could further derate the carrying capacity.

4.2 AndyMark 57 Sport

AndyMark's 57 Sport Gearboxes are rated on a per-gearbox configuration with a maximum torque capacity.

4.3 REV UltraPlanetary

REV's UltraPlanetaries have a load rating of 40 N-m at the final cartridge (output).