Documentation and Validation of EveryCalc's Belt Calculator

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May 27, 2020

Abstract

Belts are pretty easy to use and calculate the appropriate distances for. When this center distance is calculated and manufactured properly, they should not require adjustment.

1 Geometry Definition

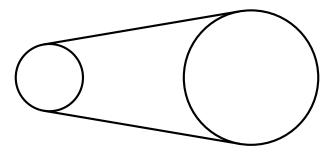


Figure 1: Belt and Sprockets

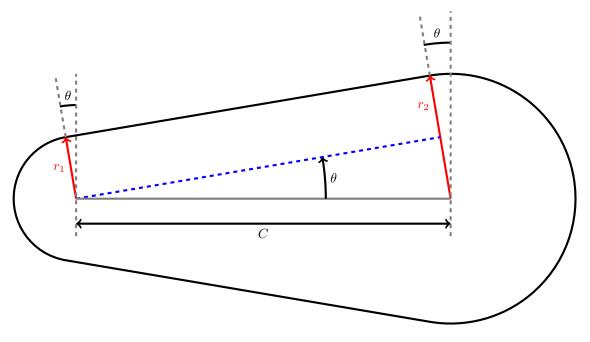


Figure 2: Belt Dimensions, Labeled

Quickly, the pitch radii and diameters of the pulleys are:

$$d_1 = 2r_1 \tag{1}$$

$$d_2 = 2r_2 \tag{2}$$

$$\sin(\theta) = \frac{r_2 - r_1}{C} \tag{3}$$

The total length of the pulley L can be expressed as:

$$L = 2 < \text{straight segment} > + < \text{arc for pulley } 1 > + < \text{arc for pulley } 2 >$$

$$L = 2\frac{C}{\cos(\theta)} + r_1(\pi - 2\theta) + r_2(\pi + 2\theta)$$
(4)

The trig identity for the cosine of an arcsine will be helpful:

$$\cos(\operatorname{asin}(x)) = \sqrt{1 - x^2} \tag{5}$$

Putting this all together lets us determine the total belt length in terms of pitch diameters d_1 , d_2 , and the center-center distance C:

$$L = \frac{2C}{\sqrt{1 - (\frac{d_2 - d_1}{2C})^2}} + \frac{d_1}{2}(\pi - 2\theta) + \frac{d_2}{2}(\pi + 2\theta)$$
(6)

This equation isn't easy to analytically solve for C in terms of d_1 , d_2 , and L. WolframAlpha yields a solution, though it is quite atrocious. I found that it's best to use a numeric algorithm (such as <u>bisection</u>, which my calculator uses).

2 Crossed Belts

The same approach can be taken with a crossed drive belt (which is used in order to reverse direction of rotation).

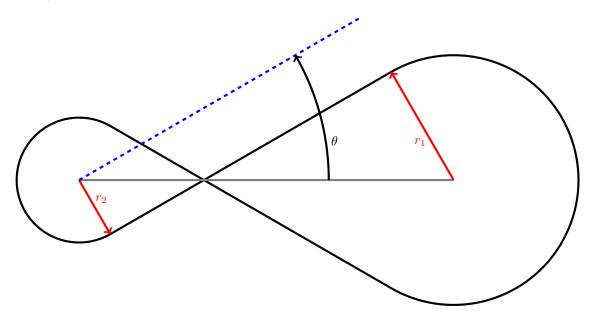


Figure 3: Belt Dimensions, Labeled

The belt angle now is

$$\sin(theta) = \frac{r_2 + r_1}{C} \tag{7}$$

$$L = 2\frac{C}{\cos(\theta)} + r_1(\pi + 2\theta) + r_2(\pi + 2\theta)$$
(8)

Resulting in:

$$L = \frac{2C}{\sqrt{1 - (\frac{d_2+d_1}{2C})^2}} + \frac{d_1 + d_2}{2}(\pi + 2\theta)$$
(9)

3 Belt Strength Calculation

Belt strength is calculated from the tables in the Gates Light Power and Precision Manual.

		r of grooves, pi								Be	elt Width	(mm)	9	15	20	25
		opriate width fa g. (See Step 4			0		tain the			Wi	idth Multi	plier	0.60	1.00	1.33	1.67
				Rated	Torque (lb-in) Fo	or Small	Sprock	et - 15n	nm Be	lt Widt	h*				
Number of Grooves	18	20	22	24	26	28	32	36	40		45	50	56	62	74	80
Pitch (mm) Diameter (in)	28.65 1.128		35.01 1.379	38.20 1.504	41.38 1.629	44.56 1.754	50.93 2.005	57.30 2.256			1.62 . 820	79.58 3.133	89.13 3.509	98.68 3.885	117.77 4.637	127.32 5.013
10 20 40 60 100	78.24 72.38 66.53 63.11 58.80	8 87.11 8 80.60 76.80	109.00 101.80 94.69 90.51 85.23	124.20 116.40 108.60 104.00 98.27	139.30 130.90 122.40 117.50 111.20	154.3 145.2 136.1 130.8 124.1	184.3 173.9 163.5 157.5 149.8	214.1 202.4 190.7 183.9 175.2	230.6 217.6 209.9	2 2 2	80.3 65.6 51.0 42.4 31.7	316.6 300.4 284.1 274.6 262.6	359.9 341.7 323.5 312.9 299.5	403.0 382.8 362.7 350.9 336.0	488.3 464.2 440.2 426.1 408.3	530.6 504.6 478.5 463.3 444.1
200 300 400 500 600	52.94 49.52 47.09 45.20 43.66	61.70 59.00 56.91	78.08 73.89 70.92 68.61 66.73	90.46 85.90 82.65 80.14 78.08	102.80 97.83 94.31 91.59 89.36	115.0 109.7 105.9 103.0 100.6	139.4 133.3 129.0 125.6 122.9	163.5 156.7 151.8 148.0 144.9) 179.7) 174.3) 170.1	2 2	17.0 08.4 02.3 97.6 93.7	246.3 236.8 230.0 224.7 220.4	281.2 270.6 263.0 257.0 252.2	315.8 304.0 295.6 289.0 283.6	384.2 370.1 360.0 352.1 345.6	418.1 402.8 391.8 383.3 376.2
Length Corre	ection Fa	actor	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15	1.20	1.25	1.30
For Belt	From	Length (mm) # of teeth	200 40	215 43	260 52	315 63	375 75	450 90	540 108	650 130	780 156	935 187	1130 226	1355 271	1625 325	1960 392
Length	To	Length (mm) # of teeth	210 42	255 51	310 62	370 74	445 89	535 107	645 129	775 155	930 186		1350 270	1620 324	1955 391	2000 400

Shaded area indicates drive conditions where reduced service life can be expected. Contact Gates Product Application Engineering for specific recommendations

Figure 4: Exemplary data from the Gates manual.

These tables list allowable pulley torque $T(\omega, N)$ as a function of RPM ω and pulley teeth N. Note that 6 teeth should be in engagement. 2-D interpolation is used to determine values on the in-betweens. Tabulated values outside the bounds are extrapolated. Omitted values are presumed to be zero.

$$T_{base} = \text{interp2D}(\{N\}, \{\omega\}, [T], N_{sprocket}, \omega_{sprocket})$$
(10)

$$K_{length} = \text{interp1D}(\{L\}, \{K_{lengths}\}, \{L_{belt}\})$$
(11)

$$K_{width} = \text{lookup}(\{w\}, \{K_{widths}\}, \{w_{belt}\})$$

$$(12)$$

$$T_{rated, sprocket} = K_{length} \times K_{width} \times T_{base}$$
⁽¹³⁾

It should also be noted that the number of teeth engaged with the pulley is recommended to be no less than 6.

4 Validation

Results of my EveryCalc belt tool have been compared to <u>West Coast Products' Belt Calculator</u>.

Case A

Belt Calculator				_	
5mm HTD/GT	~ 2 ∨				
Pulley 1		Pulley 2		Center Distance	e
Number of Teeth	24	Number of Teeth	18	Desired Center	5
Outer Diameter	1.4589	Outer Diameter	1.0830	Center Add	0.005
Pitch Diameter	1.5038	Pitch Diameter	1.1279	Ratio	1.3333
Smaller Belt		Larger Belt			
# of Teeth	70	# of Teeth	80		
Center Distance	4.8241	Center Distance	5.8090		
P1 Teeth in Mesh	12.2978	P1 Teeth in Mesh	12.2473		
P2 Teeth in Mesh	8.7767	P2 Teeth in Mesh	8.8146		

Figure 5: Case A: WCP Calculator

💿 Timing Belt 🔵 Polybelt											
Pitch:	5	[mm]									
	Pulle	ey 1	Р	ulle	ey 2			Cen	ter		
# Teeth	24		18					sired C 5		5	
OD	1.5471	1.1712		12			Center Add 0.0		005		
PD	1.5038		1.12	79						7500	
Belting Option Source:					creme	crement: 10				Teeth	
Smalle					er		Larger				
# of 1		# of T	eeth	eth 70.0000			80.0000				
Ler		ngth	13.7795			15.7480					
		enter-Ce	nter 4.8240			5.8089					
	P1 Te	eth in M	4esh	esh 12.2978			12.2473				
	P2 Te	eth in M	4esh	8.7767		8.8146					
	Gates				lculated Precision			he Jesign Ma	nu	al	
	g Series	HTD		•	9mm		-	width			
RPM 100		100			133.33	3	[RPM]				
	Torque	20			15.000		[in-lbf]				
Torque C	apacity	37.168			25.664		[in-lbf]				
Factor of	f Safety	1.711									

Figure 6: Case A: EveryCalc

		WCP	EveryCalc V0.5
	Number of Teeth	70	70
Smaller	Center-Center	4.8241	4.8240
Smaner	P1 Teeth Mesh	12.2978	12.2978
	P2 Teeth Mesh	8.7767	8.7767
	Number of Teeth	80	80
Langer	Center-Center	5.8090	5.8089
Larger	P1 Teeth Mesh	12.2473	12.2473
	P2 Teeth Mesh	8.8146	8.8146

Table 1: Case A: Comparison of results

No issues here other than minor rounding errors.

Case B

Belt Calculator	2~			-	
Pulley 1		Pulley 2		Center Distance	9
Number of Teeth	10	Number of Teeth	42	Desired Center	18
Outer Diameter	0.3460	Outer Diameter	1.5492	Center Add	0.005
Pitch Diameter	0.3760	Pitch Diameter	1.5792	Ratio	0.2381
Smaller Belt		Larger Belt			
# of Teeth	180	# of Teeth	N/A		
Center Distance	9.0794	Center Distance	N/A		
P1 Teeth in Mesh	4.7889	P1 Teeth in Mesh	N/A		
P2 Teeth in Mesh	21.8865	P2 Teeth in Mesh	N/A		

Figure 7: Case B: WCP Calculator

EveryCalc 0.5	Inches	•				
 Timing Belt Polybelt 						
Pitch: 3	[mm]					
Pulle	eyl Pu	lley 2		Cen	ter	
# Teeth 10	42		Belt 1	eeth 🛨	180	
OD	OD		Cer		0.005	
PD 0.3760	1.579	0		Ratio	4.2000	
			80.0000			
	L		1.2598			
	Center-C	enter 9.	0795			
	P1 Teeth in	Mesh 4.	7890			
	P2 Teeth in	Mesh 2	1.8864			
Gates	Strengh o Light Power &				nual	
Belting Series	GT2	- 6mm	· <u>-</u>	width		
RPM 100		133.33	33	[RPM]		
Torque			D	[in-lbf]		
Torque Capacity	4.690	30.409	30.409		[in-lbf]	
Factor of Safety	0.235					

Figure 8: Case B: EveryCalc

	WCP	EveryCalc V0.5
Number of Teeth	180	180
Center-Center	9.0794	9.0795
P1 Teeth Mesh	4.7889	4.7890
P2 Teeth Mesh	21.8865	21.8864

Table 2: Case B: Comparison of results

No issues here other than minor rounding errors.