Documentation and Validation of EveryCalc's Belt Calculator

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Abstract

Belts are pretty easy to use and calculate the appropriate distances for. When this center distance is calculated and manufactured properly, they should not require adjustment.

1 Geometry Definition

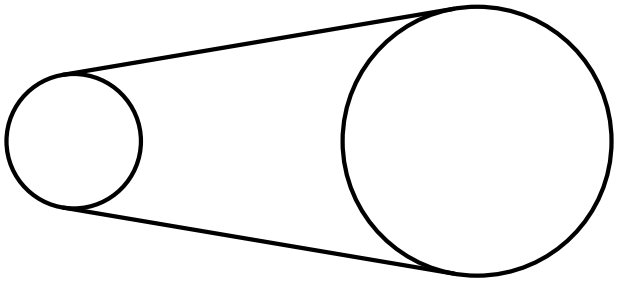

Figure 1: Belt and Sprockets

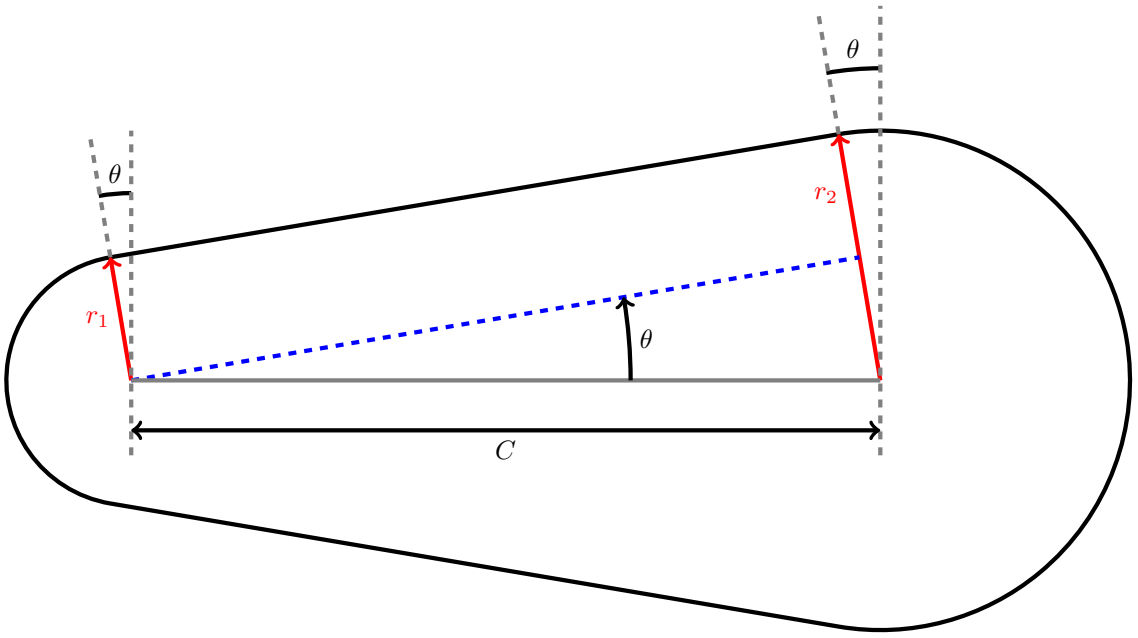

Figure 2: Belt Dimensions, Labeled

Quickly, the pitch radii and diameters of the pulleys are:

$$d_1 = 2r_1 \tag{1}$$
$$d_2 = 2r_2 \tag{2}$$
$$\sin(\theta) = \frac{r_2 - r_1}{C} \tag{3}$$

The total length of the pulley L can be expressed as:

$$L = 2 < \text{straight segment} > + < \text{arc for pulley 1} > + < \text{arc for pulley 2} >$$
$$L = 2\frac{C}{\cos(\theta)} + r_1(\pi - 2\theta) + r_2(\pi + 2\theta) \tag{4}$$

The trig identity for the cosine of an arcsine will be helpful:

$$\cos(\text{asin}(x)) = \sqrt{1 - x^2} \tag{5}$$

Putting this all together lets us determine the total belt length in terms of pitch diameters d_1, d_2, and the center-center distance C:

$$L = \frac{2C}{\sqrt{1 - (\frac{d_2 - d_1}{2C})^2}} + \frac{d_1}{2}(\pi - 2\theta) + \frac{d_2}{2}(\pi + 2\theta) \tag{6}$$

This equation isn't easy to analytically solve for C in terms of d_1, d_2, and L. WolframAlpha yields a solution, though it is quite atrocious. I found that it's best to use a numeric algorithm (such as bisection, which my calculator uses).

2 Crossed Belts

The same approach can be taken with a crossed drive belt (which is used in order to reverse direction of rotation).

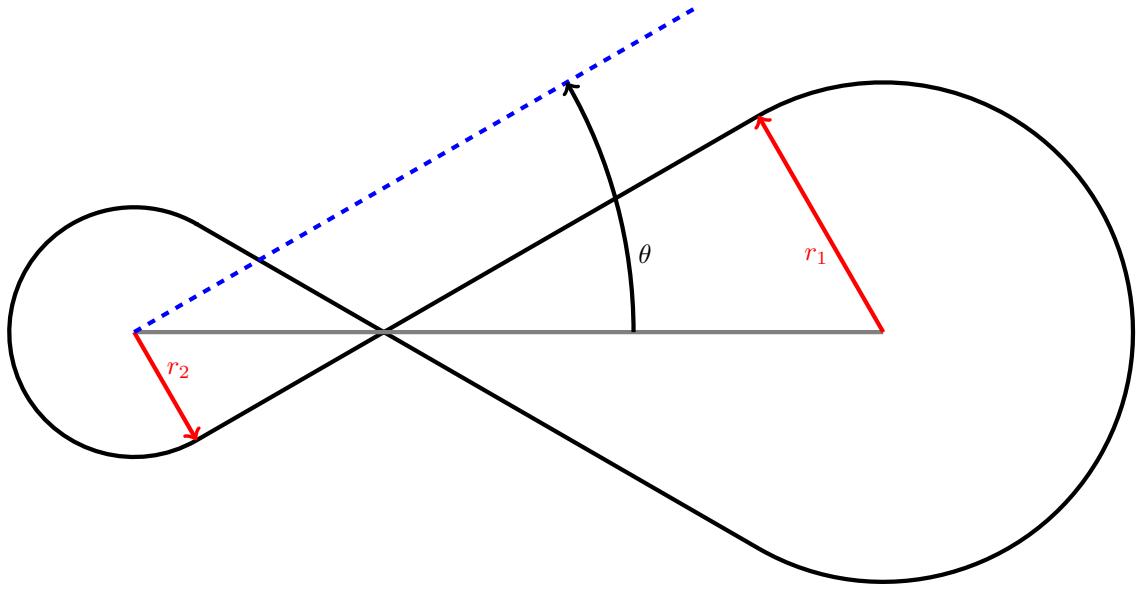

Figure 3: Belt Dimensions, Labeled

The belt angle now is

$$\sin(theta) = \frac{r_2 + r_1}{C} \tag{7}$$

$$L = 2\frac{C}{\cos(\theta)} + r_1(\pi + 2\theta) + r_2(\pi + 2\theta) \tag{8}$$

Resulting in:

$$L = \frac{2C}{\sqrt{1 - (\frac{d_2+d_1}{2C})^2}} + \frac{d_1 + d_2}{2}(\pi + 2\theta) \tag{9}$$

3 Belt Strength Calculation

Belt strength is calculated from the tables in the Gates Light Power and Precision Manual.

predetermined number of grooves, pitch diameters and rpm's. These ratings must be multiplied by the appropriate width factor and applicable belt length factor to obtain the corrected torque rating. (See Step 4 of Drive Selection Procedure on Page 24.)

Belt Width (mm)	9	15	20	25
Width Multiplier	0.60	1.00	1.33	1.67

Rated Torque (lb-in) For Small Sprocket - 15mm Belt Width*

Number of Grooves	18	20	22	24	26	28	32	36	40	45	50	56	62	74	80
Pitch (mm) Diameter (in)	28.65 **1.128**	31.83 **1.253**	35.01 **1.379**	38.20 **1.504**	41.38 **1.629**	44.56 **1.754**	50.93 **2.005**	57.30 **2.256**	63.66 **2.506**	71.62 **2.820**	79.58 **3.133**	89.13 **3.509**	98.68 **3.885**	117.77 **4.637**	127.32 **5.013**
10	78.24	93.61	109.00	124.20	139.30	154.3	184.3	214.10	243.6	280.3	316.6	359.9	403.0	488.3	530.6
20	72.38	87.11	101.80	116.40	130.90	145.2	173.9	202.40	230.6	265.6	300.4	341.7	382.8	464.2	504.6
40	66.53	80.60	94.69	108.60	122.40	136.1	163.5	190.70	217.6	251.0	284.1	323.5	362.7	440.2	478.5
60	63.11	76.80	90.51	104.00	117.50	130.8	157.5	183.90	209.9	242.4	274.6	312.9	350.9	426.1	463.3
100	58.80	72.01	85.23	98.27	111.20	124.1	149.8	175.20	200.4	231.7	262.6	299.5	336.0	408.3	444.1
200	52.94	65.51	78.08	90.46	102.80	115.0	139.4	163.50	187.4	217.0	246.3	281.2	315.8	384.2	418.1
300	49.52	61.70	73.89	85.90	97.83	109.7	133.3	156.70	179.7	208.4	236.8	270.6	304.0	370.1	402.8
400	47.09	59.00	70.92	82.65	94.31	105.9	129.0	151.80	174.3	202.3	230.0	263.0	295.6	360.0	391.8
500	45.20	56.91	68.61	80.14	91.59	103.0	125.6	148.00	170.1	197.6	224.7	257.0	289.0	352.1	383.3
600	43.66	55.19	66.73	78.08	89.36	100.6	122.9	144.90	166.7	193.7	220.4	252.2	283.6	345.6	376.2

Length Correction Factor			0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15	1.20	1.25	1.30
For Belt Length	From	Length (mm)	200	215	260	315	375	450	540	650	780	935	1130	1355	1625	1960
		# of teeth	40	43	52	63	75	90	108	130	156	187	226	271	325	392
	To	Length (mm)	210	255	310	370	445	535	645	775	930	1125	1350	1620	1955	2000
		# of teeth	42	51	62	74	89	107	129	155	186	225	270	324	391	400

Shaded area indicates drive conditions where reduced service life can be expected. Contact Gates Product Application Engineering for specific recommendations.

Figure 4: Exemplary data from the Gates manual.

These tables list allowable pulley torque $T(\omega, N)$ as a function of RPM ω and pulley teeth N. Note that 6 teeth should be in engagement. 2-D interpolation is used to determine values on the in-betweens. Tabulated values outside the bounds are extrapolated. Omitted values are presumed to be zero.

$$T_{base} = \text{interp2D}(\{N\}, \{\omega\}, [T], N_{sprocket}, \omega_{sprocket}) \tag{10}$$
$$K_{length} = \text{interp1D}(\{L\}, \{K_{lengths}\}, \{L_{belt}\}) \tag{11}$$
$$K_{width} = \text{lookup}(\{w\}, \{K_{widths}\}, \{w_{belt}\}) \tag{12}$$
$$T_{rated,\ sprocket} = K_{length} \times K_{width} \times T_{base} \tag{13}$$

It should also be noted that the number of teeth engaged with the pulley is recommended to be no less than 6.

4 Validation

Results of my EveryCalc belt tool have been compared to West Coast Products' Belt Calculator.

Case A

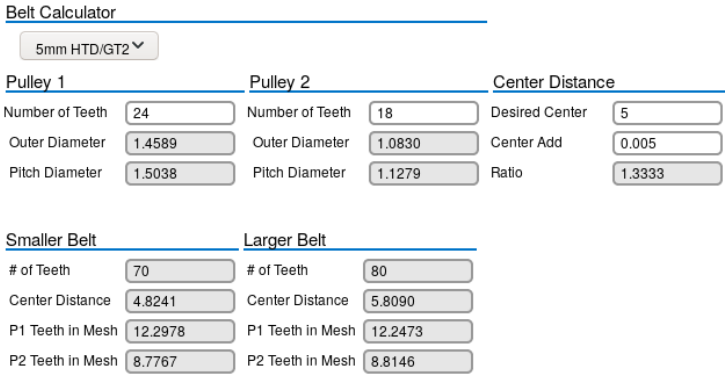

Figure 5: Case A: WCP Calculator

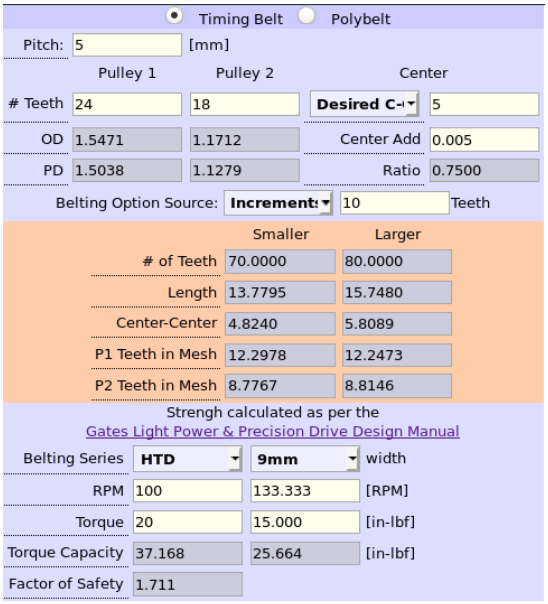

Figure 6: Case A: EveryCalc

		WCP	EveryCalc V0.5
Smaller	Number of Teeth	70	70
	Center-Center	4.8241	4.8240
	P1 Teeth Mesh	12.2978	12.2978
	P2 Teeth Mesh	8.7767	8.7767
Larger	Number of Teeth	80	80
	Center-Center	5.8090	5.8089
	P1 Teeth Mesh	12.2473	12.2473
	P2 Teeth Mesh	8.8146	8.8146

Table 1: Case A: Comparison of results

No issues here other than minor rounding errors.

Case B

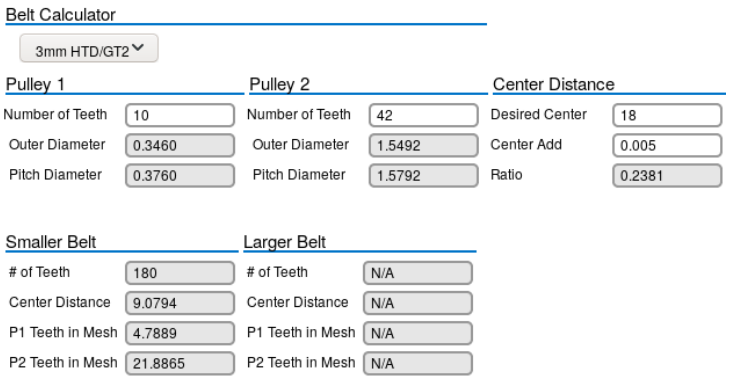

Figure 7: Case B: WCP Calculator

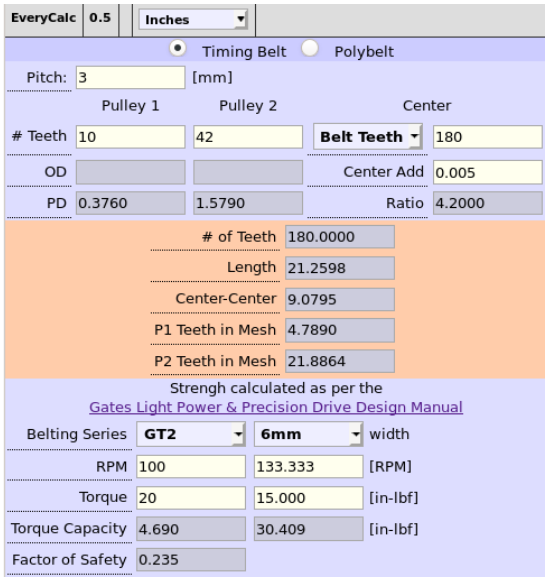

Figure 8: Case B: EveryCalc

	WCP	EveryCalc V0.5
Number of Teeth	180	180
Center-Center	9.0794	9.0795
P1 Teeth Mesh	4.7889	4.7890
P2 Teeth Mesh	21.8865	21.8864

Table 2: Case B: Comparison of results

No issues here other than minor rounding errors.